

SIMULATION STUDY OF THE DRIVER'S CONTROL  
FOR SUDDEN CHANGES IN PREVIEWED PATH

by

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## SUMMARY

Experiments were performed using three different types of visual simulations of automobile driving: 1) an abstract spot moving on an oscilloscope; 2) a television picture resulting from a TV camera driven by a model car through a model environment; 3) an actual car driven through a test course. The driver subject in each case was asked to steer his vehicle so that its trajectory would successively coincide with one or more targets which suddenly appeared in his view ahead in pseudo-random positions. Forward velocity was not under the subject's control and was fixed. Forward velocity, steering dynamics, and display-control geometrical relationships were comparable among the three control tasks.

Experimental questions concerned: 1) the effect of preview time, the time during which the driver could view the input before his vehicle's response to that part of the input was critical; 2) the effect of configuration of targets--their number, the a priori uncertainty of their positions, and their relative spacing--and especially how a second target affects response to a first target; 3) the variability of trajectories for the same target configuration and preview time; 4) the degree to which the human responds optimally with respect to a performance criterion, given or inferred.

Results with the TV remote-controlled model car simulator showed that the subject responded later than in the computer oscilloscope display or in the actual car experiments. With both the actual car and TV simulation he tended to respond to each target as it came. By contrast in the less realistic or more abstract scope display experiment the subject tended to "get lined up" ahead of time with respect to the line connecting the two targets. These differences may have been due to the fact that the scope targets were not seen in perspective whereas the model TV and actual car targets were necessarily viewed in perspective. Further, the TV simulator constrained peripheral vision. In general the subjects consistently differed from optimal control performance based on a criteria implied by the subjects' instructions, the difference being characterized by

insufficient planning for the second of two targets until after engagement with the first. With training there was some trend in the direction of optimal performance.

Of the three experiments inter-run variability was greatest in the TV model car simulator, least in the scope simulator. Variability was usually greater for the second of two targets, though the variance ratio was not statistically significant ( $>6.39$  for  $df = 4/4$ ).

In the TV simulator when two spatially ordered targets were presented simultaneously the variability of response to the second target was greater than when presentation of the second target was delayed by one half second. Again the variance ratios were not statistically significant ( $>6.39$  for  $df = 4/4$ ).

Some results for experiments in which the driver was forced to steer for one target (of two possible positions) in open-loop fashion (without any visual feedback) after a single glance at the actual target position showed surprisingly little difference from the closed-loop results.

#### ACKNOWLEDGMENTS

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This report is a revision of a master's thesis in Mechanical Engineering by the first author and supervised by the second. The thesis "Experimental Study of Preview Control in a Simulated Driving Task" was submitted in August 1966.

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## 1. INTRODUCTION

Driving simulation usually involves presenting an artificial representation of a driving environment to a human subject. The fidelity of the presentation required depends on the objective of the experiments. In the past, emphasis in driving simulation has been on training drivers. More recently simulators have been used for research into those aspects of driving normally involving hazard to the driver.

Designs for simulators range from moving base simulators with elaborate visual displays to simplified part-task simulators whose purpose is the study of a single variable. Among the techniques presently employed for visual simulations of driving are: 1) prefilmed moving pictures whose frame rate and position relative to the viewer may be moved slightly during the simulator run<sup>1</sup>; 2) scale models on moving belts viewed through enlargement optics<sup>2</sup> or closed circuit television; 3) closed circuit TV carried by model cars and driven along model roadways<sup>4</sup>; 4) shadowgram projections of objects driven relative to a fixed point light source and viewed from the other side of a fixed translucent screen<sup>5</sup>; 5) oscilloscope traces generated by analog or digital computers<sup>6,7,8</sup>. Motion cues are sometimes added through use of tilting or vibrating seats though the latter have yet to be proven. Unfortunately, there is little known about how to evaluate the degree of fidelity necessary for effective simulation for any given purpose. Recently some research has been directed toward studying the validity of simulators.

It would be desirable to formulate a model of the driver which would predict the driver's response for various traffic configurations, states of the driver, etc. thus allowing the researcher to anticipate those crucial situations with high accident probability. The classical servo-mechanism model of the human operator has been applied successfully

to continuous tracking tasks<sup>9</sup>. Attempts have been made to extend the usefulness of the automatic control theory models from simple present-state error-nulling to situations like automobile driving by incorporating into these models the ability to "look ahead" or preview<sup>10,11,12</sup>. This ability to preview the input and plan ahead is inherent in much human motor skill behavior. Sometimes preview of one input is time shared with other inputs in which case probabilistic sampling must be included in the model<sup>13</sup>.

Sheridan suggested three models of preview control<sup>14</sup>. His first model is an extension of the normal linear convolution integral to operate on the input prior to the time a response is required. His second and third models presume that the human incorporates a fast-time dynamic analog of himself and the controlled process. His third model determines an optimal trajectory on the basis of the previewed input, where the trajectory is optimal with respect to a specific error-effort penalty tradeoff presumably inherent in the operator.

In a paper by Sheridan and Roland<sup>15</sup> this optimal control model is defined in terms of the driving situation. The optimal control strategy is the minimization of the cost function or trading relation between the penalty for colliding with obstacles (function of position) and the cost of increased effort (function of steering, braking, and acceleration).

The present research was an effort to study the behavior of the human operator in driving simulation experiments of three degrees of realism:

- 1) A scale model remote controlled car which carried a closed circuit TV camera was driven through a model environment. This served as a fixed-base visually realistic driving simulator. Attention was paid to possible effects on the driver's behavior due to the psychological refractory period<sup>16</sup> and to the information content<sup>17</sup> of the stimuli.

Further description of the simulations and experiments is presented in Sections 2,3, and 4.

2) A computer generated oscilloscope display experiment was employed to provide a more abstract simulation of the same tasks performed in the visually realistic experiments described previously. This particular experiment was a follow up from early study<sup>18</sup>. This oscilloscope display experiment was designed to have the same control dynamics and task constraints as an optimal control model also programmed by the author. Thus a direct comparison could be made between the optimal trajectories and the trajectories of the human operator for equivalent tasks. The optimal control model is based on the dynamic programming algorithm of Bellman<sup>19</sup>. This simulation and these experiments are described in Sections 5 and 6.

3) A standard American automobile was used for control tasks similar to the above two simulation experiments. These experiments are described in Sections 7 and 8.

## 2. DESCRIPTION OF EXPERIMENTAL APPARATUS AND PROCEDURE

### FOR T.V. REMOTE CONTROLLED CAR SIMULATION

#### Simulator Equipment

The simulation consists of a scale model car which carries a TV camera and is steered, accelerated, and braked through a scale model environment by a human subject. The subject sits in a fixed booth, views on a TV monitor what the camera sees, and uses a conventional steering wheel, accelerator, and brake to control the scale model car. The dimensions and other pertinent specifications of the remote controlled car, Figures 1-4, are given in Table 1.

The car's forward motion is powered by a d.c. motor. Within the control booth, the accelerator pedal is connected through a linkage to an auto-transformer. The rectified voltage from the auto-transformer is the input to the d.c. drive motor on the car. The gear train between the drive motor and the rear wheels contains an electro-magnetic clutch-type brake. A potentiometer connected to the brake pedal linkage in the control booth determines the stopping force. The spring rates of both the brake and accelerator pedals were designed with reference to actual automobiles.

length of wheel base	8.75 inches
width of wheel base at front axle	5.0 inches
width of wheel base at rear axle	6.0 inches
overall length	16.5 inches
overall width	7.5 inches
overall height	7.3 inches
minimum turning radius	18.5 inches
maximum speed	8.2 feet per second
effective height of T.V. lens above driving surface	2.5 inches
field of view	29 degrees
Table 1. Specifications of Remote Controlled Car	

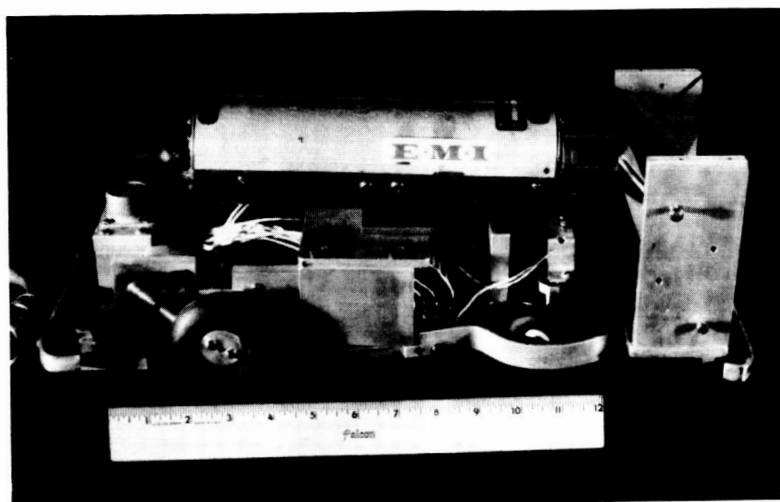


Figure 1. Remote Controlled Car

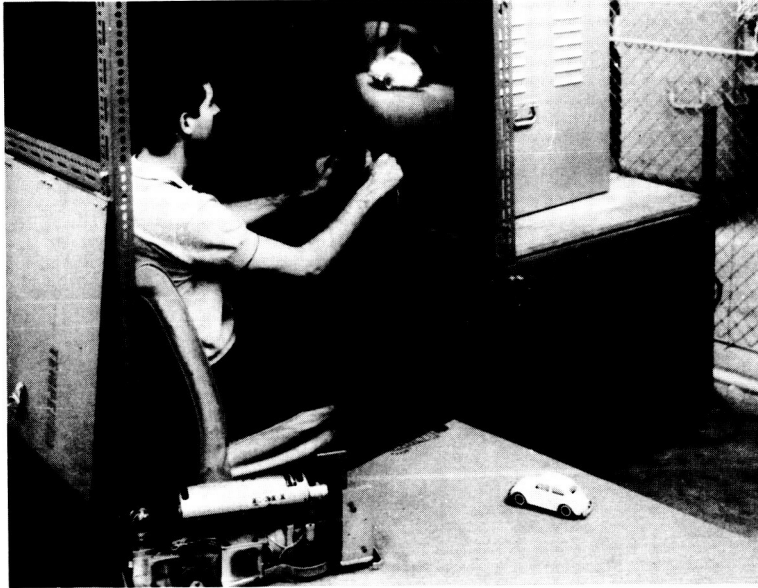


Figure 2. Control Booth and Remote Controlled Car



Figure 3. Subject Viewing Targets on T.V. Monitor

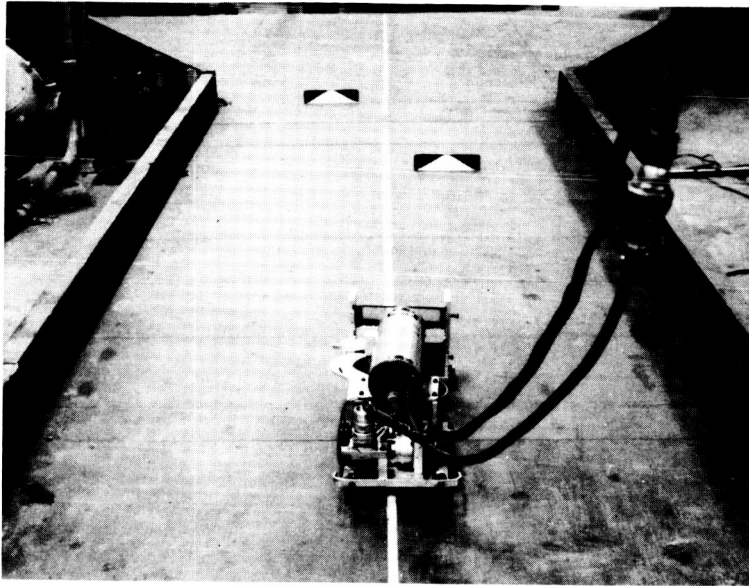


Figure 4. Remote Controlled Car Attempting to Hit Targets on Test Track

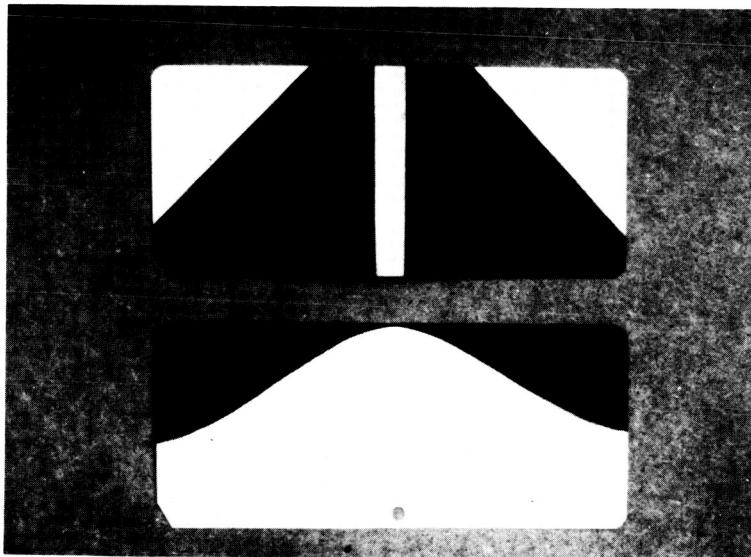


Figure 5. Two Types of Targets

The steering device consists of a motor-generator set on the car which turns the front wheels through a rack and pinion linkage and provides velocity feedback to the servo-amplifier. The balancing potentiometers of the servo-system are attached to the steering wheel shaft in the control booth and to the rack on the front wheels of the car. In an actual automobile the force on the steering wheel is approximately proportional to the rate of turn of the automobile. However, because of the complexity involved in providing such feedback for the simulator, the spring rate of the steering wheel is proportional to its angle of turn.

The television camera is mounted on the car above the other equipment with its axis horizontal and forward, Figure 1. Because of the low ambient light level and the insensitivity of the camera, it is necessary to operate the camera with the lens opened to  $F/1.3$ , thus greatly reducing the effective depth of field. Mounted on the car in front of the television camera are two 2 inch by 5 inch mirrors. The orientation of the mirrors is such that the driver sees the track ahead from the normal elevation. Without the mirrors the view of the track appears to be that of looking down from about twenty feet above the driving surface. The car has spring loaded bumpers on the front and rear to prevent damage to the equipment on the car due to crashes into the side walls of the track.

Centered between the front wheels and directly behind the mirrors is a small solenoid-actuated pen mechanism for marking the trajectory of the car on paper laid on the track surface. The solenoid is controlled by the relay switching circuitry and the pen is in the marking position only during the interval that the target is present. The paper on which the trajectories are recorded is flat black so that it will not cause a glare on the television. However, the soft pencil lead used in the pen mechanism will draw a legible trajectory on the paper.

The television monitor and controls for the car are mounted in a control booth, Figure 2. The booth is approximately 3 feet wide, 5 feet long, and five feet high. The driver may be totally enclosed within the booth to shut out external light, thus providing improved viewing of the television. An adjustable automobile seat is mounted on the floor of the booth. The steering wheel, brake pedal, and accelerator pedal are mounted in the same positions as on American automobiles. The twenty-one inch wide television screen is mounted at eye level, twelve inches behind the top of the steering wheel. The television screen is normally at a viewing distance of approximately twenty-eight inches from the driver. The lower front portion of the control booth contains the servo-amplifier for the steering servo-system, the foot pedal controlled auto-transformer and rectifier for the drive motor, and the auxiliary auto-transformer for the drive motor. The purpose of the two separate auto-transformers for the drive motor is that it is desirable for the driver to have control of his speed during his return to the starting position for each run; however, during the run it is a necessary part of the experimental design that the speed remain constant or as nearly so as practical. The switch from the driver controlled auto-transformer to auxiliary auto-transformer with a preset voltage is made by the electric eyes and the relay switching circuitry.

The car is driven on a wooden surface 45 inches wide and 48 feet long, Figure 4. The car is prevented from running off the driving surface by six-inch high wooden walls on both sides and spring loaded restraining wires at the ends of the track.

The section of track immediately preceeding the flanged section has seven crosswise slots 12 inches apart. The slots are one sixteenth of an inch wide and span the entire width of the track. There is an eighth slot in the flanged section of track 12 inches from the last slot in the previous section of track. Targets made of stiff 3 inch by 8 inch paper cards emerge from the slots at the proper time during a run. A solenoid

driver mechanism quickly elevates the target from below to above the track surface. The target mechanism is mounted on a slider below the track so that the crosswise position of the card in the slot may be changed as per the experimental plan. Also the slider may be removed and placed beneath any of the slots. To present two targets simultaneously or in sequence, several experiments require the use of two such mechanisms.

The first thirty-two feet of the track is equipped with electric eyes or light dependent resistance (LDR) devies spaced at two foot intervals. In the section of track where the targets are presented the electric eyes are spaced at one foot intervals. Each electric eye unit consists of a 6 volt lamp and focusing lens on one side of the track aimed at a light dependent resistance on the opposite side of the track. The lamps and L.D.R.'s are at such a level that the leading edge of the car will interrupt the light beam and signal the relay switching circuitry. Each L.D.R. may be connected to one of eight relay circuits. These relay circuits are used together as switches for the clocks, solenoids, and auto-transformers. Eight relay circuits are mounted in modules of four each together with power supplies. All the pertinent terminals of each relay circuit are connected to pin jacks on the back of each module. This patchboard design allows the experimenter flexibility in designing and changing switching circuits for various experiments.

The television cables and control cables from the car are suspended by a trolley centered seven feet above the track. The purpose of the trolley is to eliminate the effect of the weight of the cable on the dynamics of the car. In addition, during an experimental run it was necessary to pull the trolley by hand, maintaining slack in the electrical umbilical cord, thus preventing its loading the vehicle.

To give some idea of the dynamics of the complete system some step input "calibration" data are presented in Appendix I.

#### Operation Procedure

The following is an outline of the operation procedure:

- 1) The driver started each run at the far end of the track and drove toward the opposite end of the track attempting to stay on the center line until a target appeared.
- 2) During a run the driver had no control over his speed. His speed was constant for a given set of runs.
- 3) \*When a target emerged from the track, the driver attempted to hit it on center with no regard for where the car was before or after he hit the target. In other words, he was not required to return to the center line nor was he required or requested to avoid hitting the side walls. It is for this latter reason that the track was flanged out at the end.
- 4) In the cases of the experiment where two targets appeared the driver was to attempt to hit both targets with equal consideration, and with no regard for where the car was before, between, or after the targets were hit.
- 5) A characteristic time was measured for each run to determine if the speed was constant over a set of runs.
- 6) The trajectory of each run was recorded from the point at which the target appeared until the car hit the last target.

The sequence of events as the car moved from start to finish during a run was as follows: After having turned the car around on the far end of the track, the driver began to slowly drive down the center line. Upon reaching point B, on Figure 6, the auxiliary auto-transformer for the drive motor was switched in and from this point the car traveled at a preset speed without the driver having acceleration control. At point C, thirty-five and one-half feet from point B,

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\*Because of the relative complexity of simulating the situation in which the driver attempted to avoid hitting objects and the difficulty in measuring his performance for this task it appeared more appropriate to design experiments such that the driver attempted to hit the obstacles which appeared in his path. The theory of hitting the obstacles is that the driver considers the targets as "holes" in a traffic pattern and is attempting to pass through the "holes". In all the present experiments the "holes" considered are stationary.

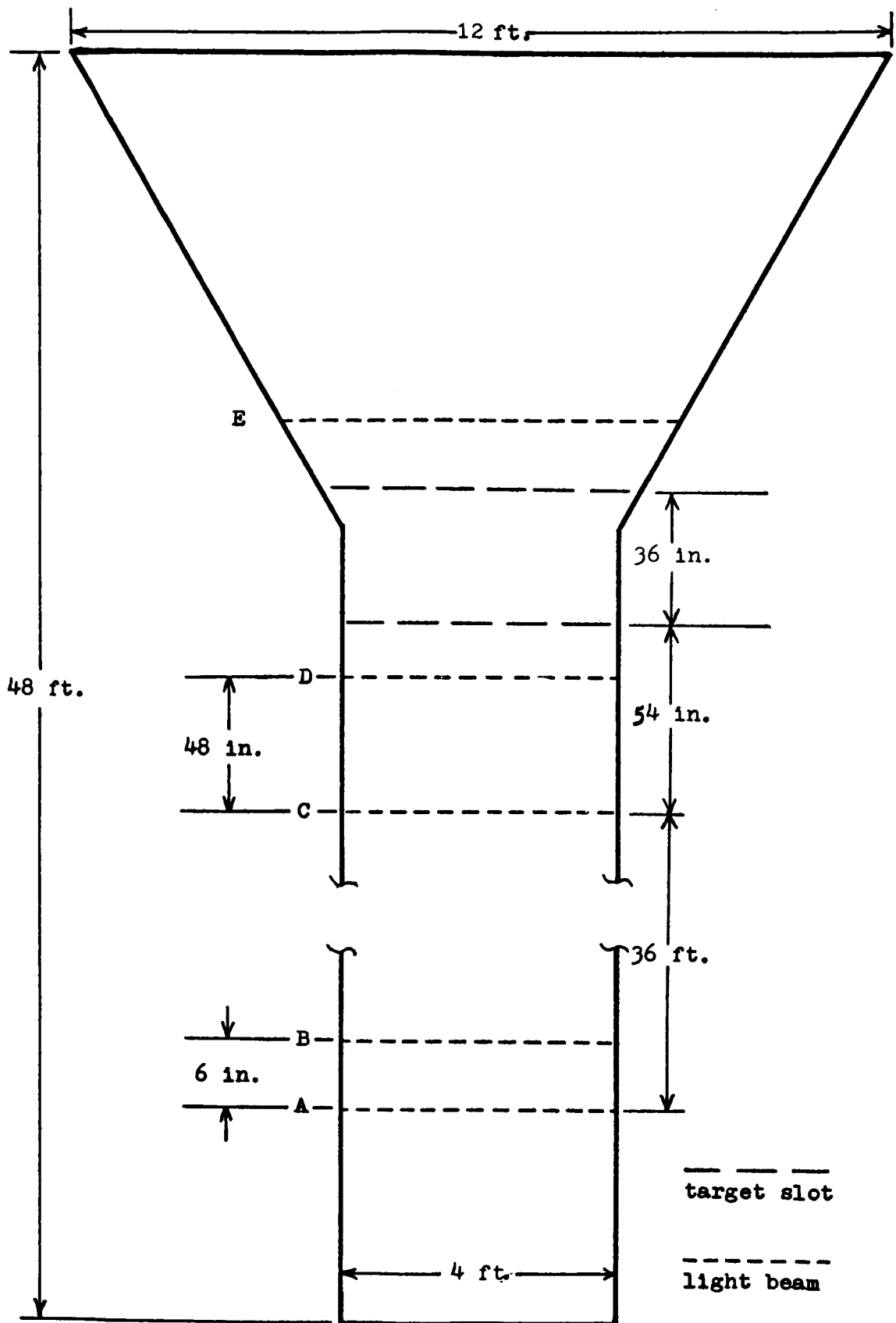


Figure 6. Plan View of Driving Simulator Test Track

the targets emerged, the clock started, and the pen began to mark the trajectory. The clock was stopped at point D. After passing over the targets the car interrupted the light beam at point E, which essentially turned off all the electric eye controlled equipment. This essentially was kept in the off state until the car returned to the end of the track. The car then interrupted the beam at point A, which reset all the relay circuitry to the "pre-run" state. The purpose of this was that the car would not cause a change in the timer or mark on the trajectory recording paper during this return to the starting position.

### 3. DESCRIPTION OF TARGET CONFIGURATIONS AND SUBJECT'S TASK FOR T.V. REMOTE CONTROLLED CAR SIMULATION

Experiments were classed according to the nine categories of target configurations called 000, 200, 300, 400, 500, 600, 700, 750, 800. These are summarized in Figure 7 and explained in detail below.

Five data runs were made for each of the different target configurations in Series 000 through Series 600. Ten runs were made for each target configuration in Series 700, Series 750, and Series 800. Each series of runs was completed as a group before the next series was started. The following description of the different series is in chronological order.

All runs on the simulator were made with the vehicle moving at a constant speed of approximately four feet per second. The accelerator foot pedal had no effect on the speed of the vehicle during the run. Runs were aborted when speeds were not within five percent of four feet per second or when externally induced noise interfered with the T.V. picture. This included up to thirty percent of the runs.

The subject's instructions were to steer the front center of the vehicle over the center of the target. The target used in Series 000 through Series 600 is the upper target of Figure 5. For runs included Series 750 and Series 800 the center of the target was precisely defined by a half inch wide vertical stripe, lower target, Figure 5.

One subject, a male M.I.T. graduate student, was used for all the experiments with the T.V. simulator. Previous to any data runs the subject was given several hours of practice on different days to familiarize himself with the simulator. The subject was given approximately ten to fifteen minutes of practice before starting data runs.

In all runs made on the driving simulator there was equal probability of occurrence of the targets at each of the possible target positions.

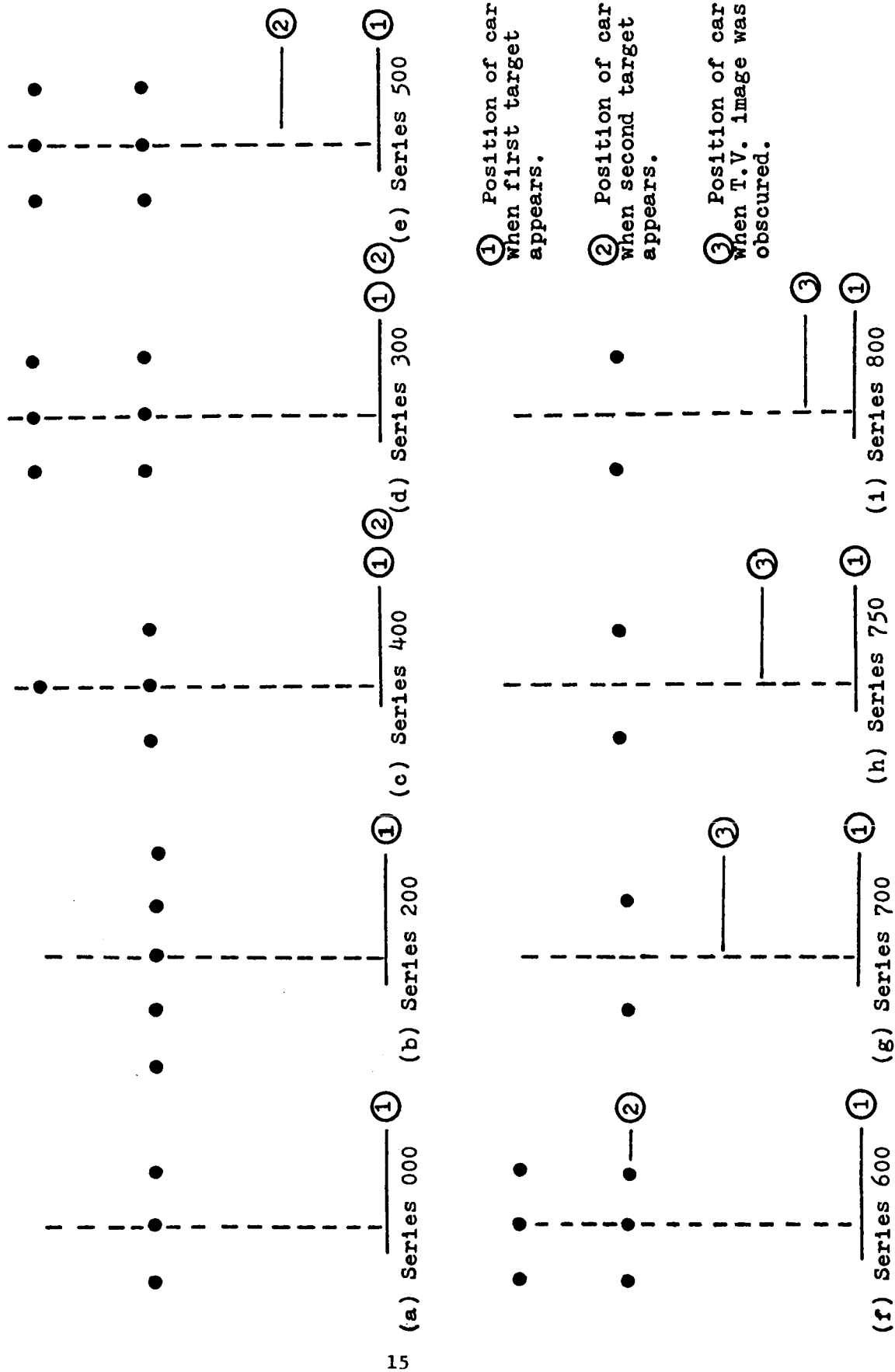


Figure 7. Target Configurations and Order of Appearance

The purpose of equal probability of target positions was to prevent the subject from developing conscious or subconscious anticipation of the appearance of the target at a particular position.

### 3.1 Series 000: One of three targets

To serve as an experimental control and to help in interpreting the results of the more interesting target configuration, the first series of runs employed the simplest possible configuration, Figure 7. A single target appeared at one of three possible positions. All three positions were fifty-four inches from the point of first sight. The three positions were: on the center line, four inches to the right and four inches to the left of the center line. Five data runs were made for each target position after about the same number of practice runs had been made. The resulting variance of those trajectories with the target on the center line establishes a norm for the variance of trajectories when the targets were located either side of the center line.

### 3.2 Series 200: One of five targets

This series of runs had five possible target positions. All five positions were fifty-four inches from the point of first sight. The five positions were: on the center line, four and eight inches to the right, and four and eight inches to the left of the center line, Figure 7. One may say that increasing the number of possible target positions from three in Series 000 to five in Series 200 increased the information content of the stimuli from 1.6 bits to 2.4 bits, where information is  $\log_2$  times the number of equiprobable alternatives. The purpose of the Series 200 runs was to observe the possible change in the trajectories for the targets four inches from the center line due to a change in the information content of the stimuli, i.e. the greater uncertainty of the target position.

3.4 Series 300: One of three targets followed by a one of three targets, both previewed simultaneously

The Series 300 target configurations consisted of two targets each of which had three possible positions. These three positions were: on the center line, four inches to the right and four inches to the left of the center line, Figure 7. As in all two target runs the first target was fifty-four inches and the second target was ninety inches from the point of first sight of the targets.

3.5 Series 500 and Series 600: One of three targets followed by one of three targets, the second appearing after the first

The driver's response to two successive stimuli within a brief time period is known to be different from the simple succession of two single target responses. The hypothetical inability of the human to respond to the second of two successive discrete stimuli within a time interval of approximately one-half second is known as the psychological refractory period. If the driver is still responding to a previous stimulus within his refractory period, response to a second stimulus may be masked or delayed.

Two sets of tasks were developed based on this hypothesis. The target positions of Series 500 and Series 600 were similar to those of Series 300. However, in the Series 500 runs the second target was delayed by .50 seconds. The second target of the Series 600 runs appeared as the vehicle crossed the location of the first target, approximately 1.12 seconds after the first target appeared in view.

Thus, if in Series 300 the driver responded as though he were receiving two temporally successive target stimuli (actually they appeared simultaneously) and the psychological refractory period obtained, one would not expect to see any difference in the trajectories of Series 300 and 500. For comparison the trajectories of Series 600 prevented the driver from initiating a response to the second target until after the first was completed.

### 3.6 Series 700, Series 750, and Series 800: One of two targets with vision occluded after short presentation

Closed loop control, when referred to manual tracking behavior means that the human continually receives information regarding the value of the error between the input and the controlled response. Close loop preview control means the human receives information about the future input (future in the sense that the vehicle response is to match this input at a future time) and from knowledge of the present position, velocity, etc. of the vehicle can predict the future error to some extent. Open loop control means he executes his response after a short visual sample of the location of the target without further visual feedback or affect on response of further visual stimuli. We hypothesize that driving behavior is a combination of open loop responses and closed loop tracking.

The tasks of Series 700, 750, and 800 measure the degree to which the driver is performing open loop. After presenting the target for a fraction of a second the T.V. image was obscured and the subject had to drive the remaining distance to the target entirely open loop.

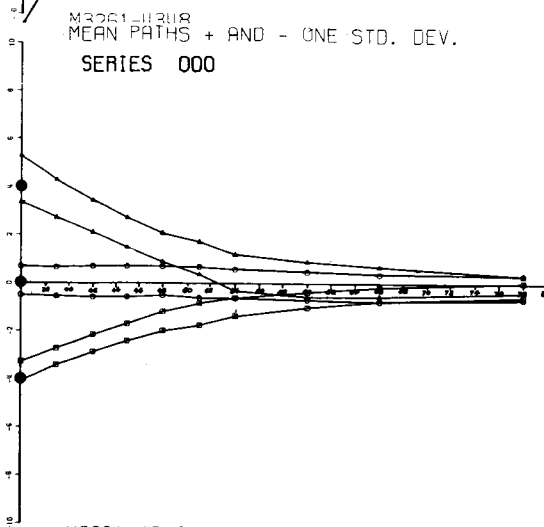
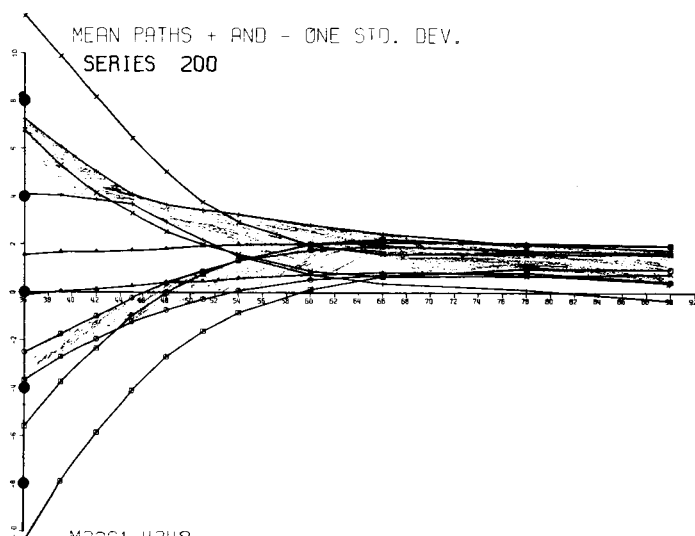
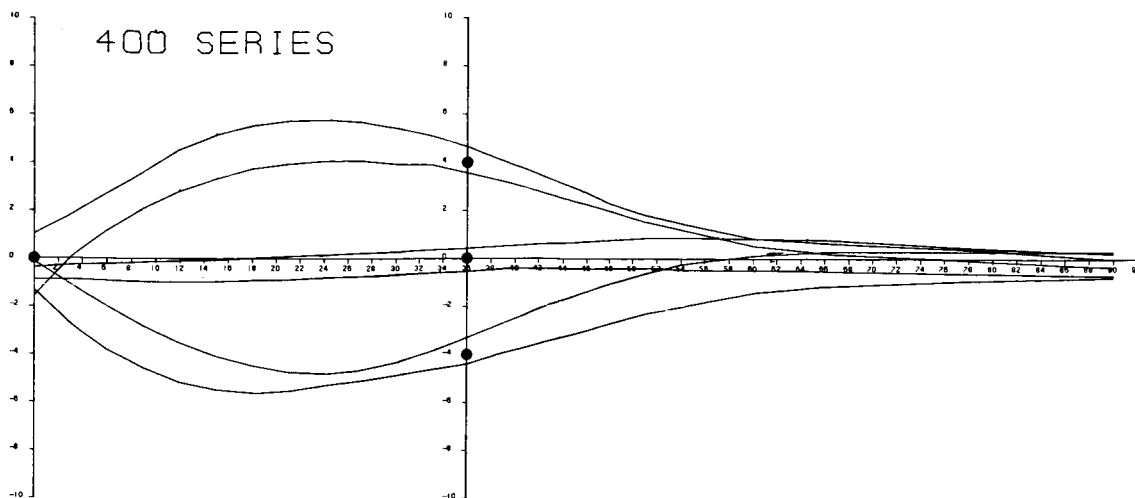
Trajectories were recorded for three occlusion times, each time in a separate set of runs. The occlusion times were .50 seconds for Series 700, .75 seconds for Series 750, and 1.00 seconds for Series 800. The target configuration, identical for the three series, was a single target with equal probability of being four inches to the right or four inches to the left of the center line. This is the simplest possible configuration which does not introduce a right or left bias. As in the previous tasks the target was fifty-four inches from the point of first sight. Each series consisted of ten runs during which the image was obscured, intermixed with ten runs with no occlusion period. The subject was told before each run whether or not the lens was to be closed. It was thought that if the subject was not told, he would always prepare himself in advance for the more difficult of the two types of runs, presupposing there was a difference in difficulty.

In this series of experiments the effective shape of the target was changed for some runs. With x appended to the series number it was the broad normal curve of the previous task. For other runs (with 0 appended) it was changed to a half inch wide vertical stripe. It was thought that if the subject's driving behavior was essentially closed loop his performance should improve over that for the broader target as a consequence of the more well defined target.

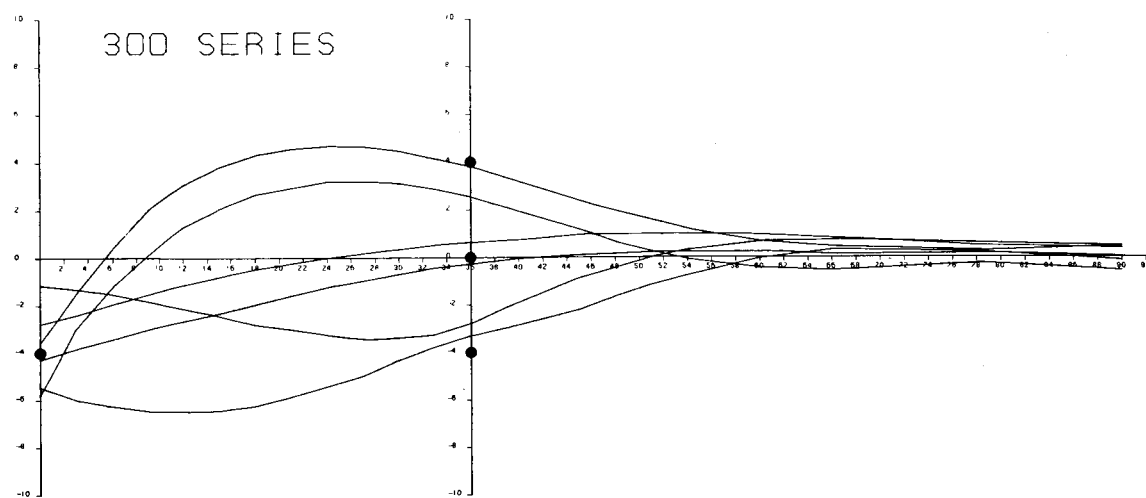
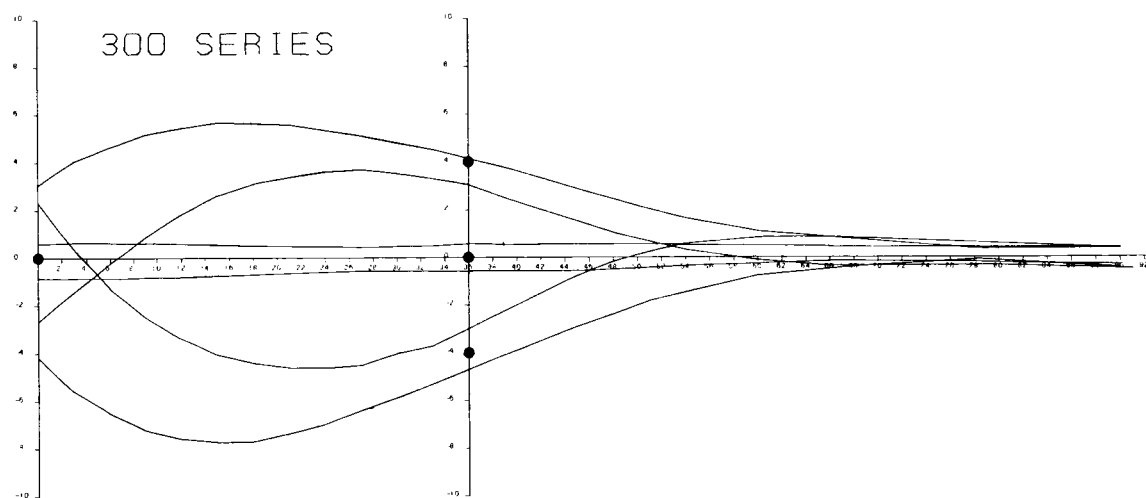
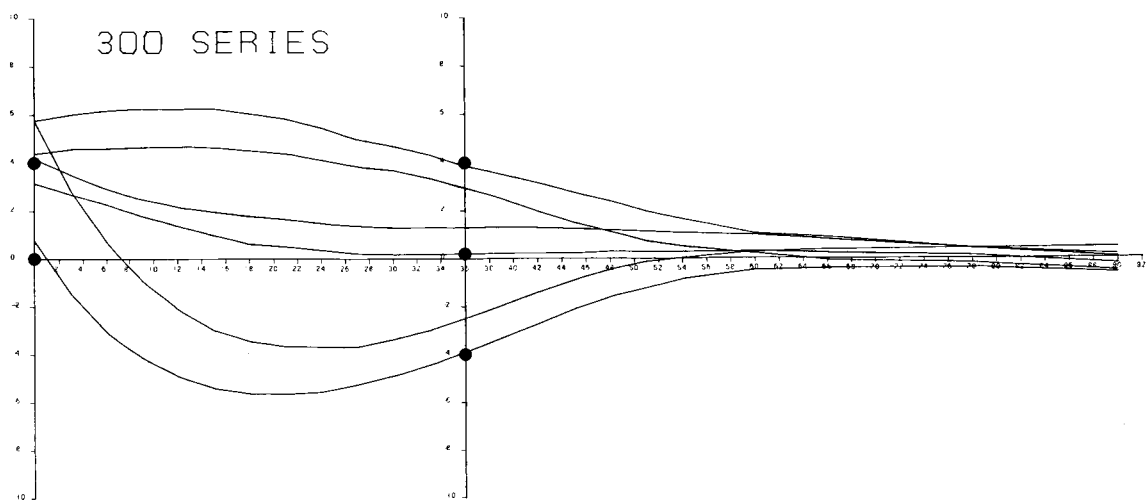
#### 4. RESULTS OF T.V. REMOTE CONTROLLED CAR SIMULATION

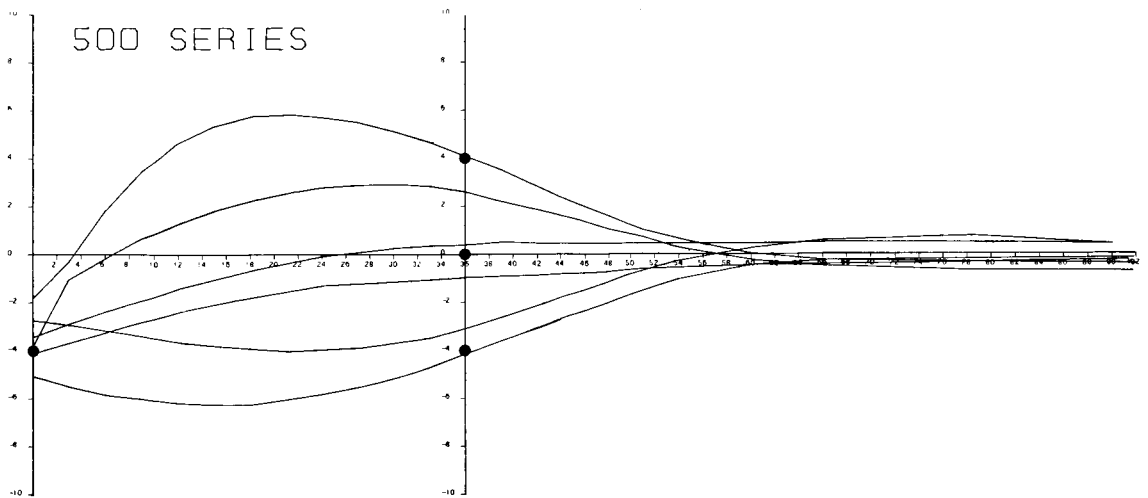
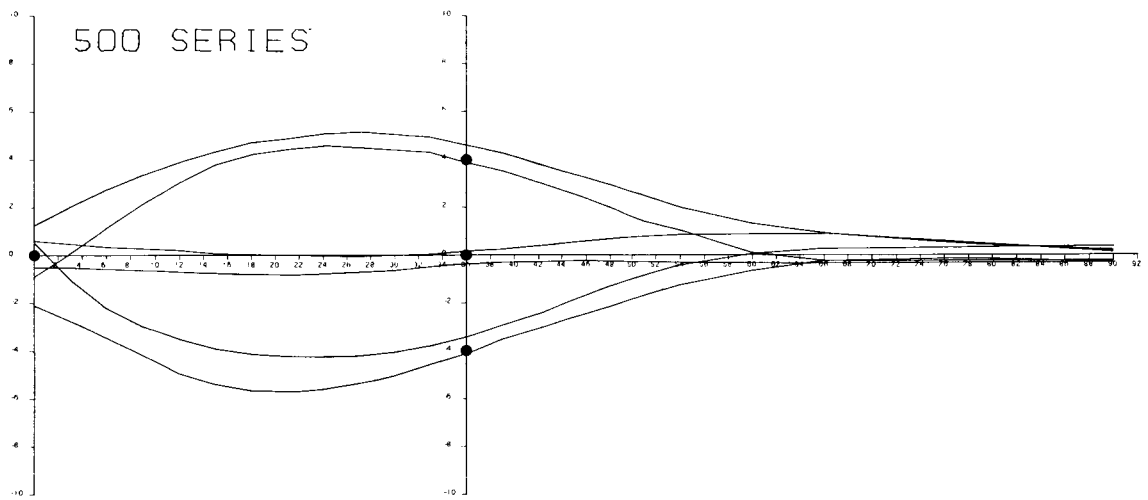
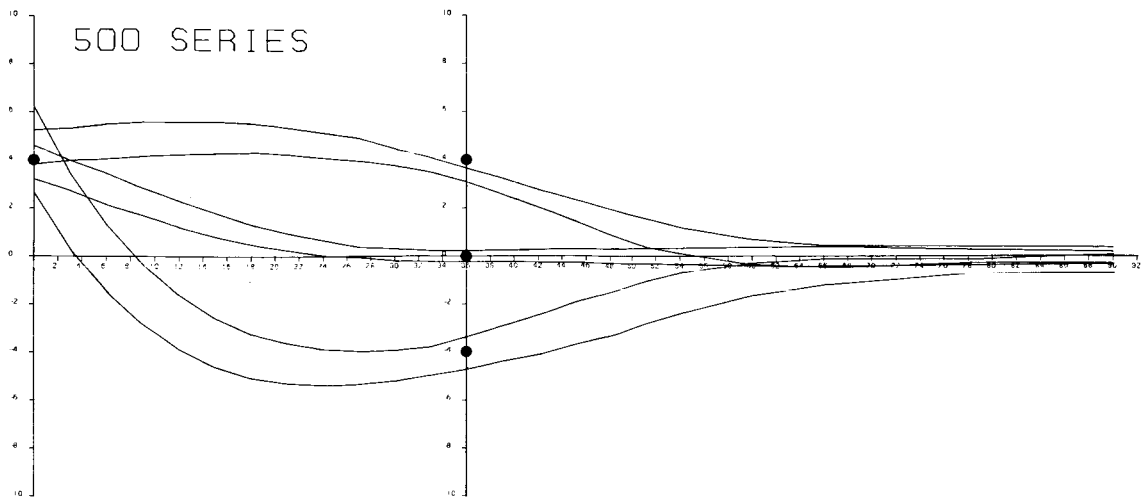
The results of the driving simulation experiments are shown on the following pages in graphical form. The curves represent the confidence limits of the trajectories of the vehicle. Each pair of curves represents one standard deviation either side of the mean of the trajectories for a particular task. Although all possible target configurations of each series were intermixed for presentation to the subject during the experiment, the curves appropriate to each target within the set of alternatives are separately presented here. Each figure shows the results of three target configurations of a single series: all three positions of the first target combined with one of the three positions of the second target. The graphs for Series 000, 200, 700, 750, and 800 shown confidence limits of one standard deviation for the corresponding two, three, or five single target configurations.

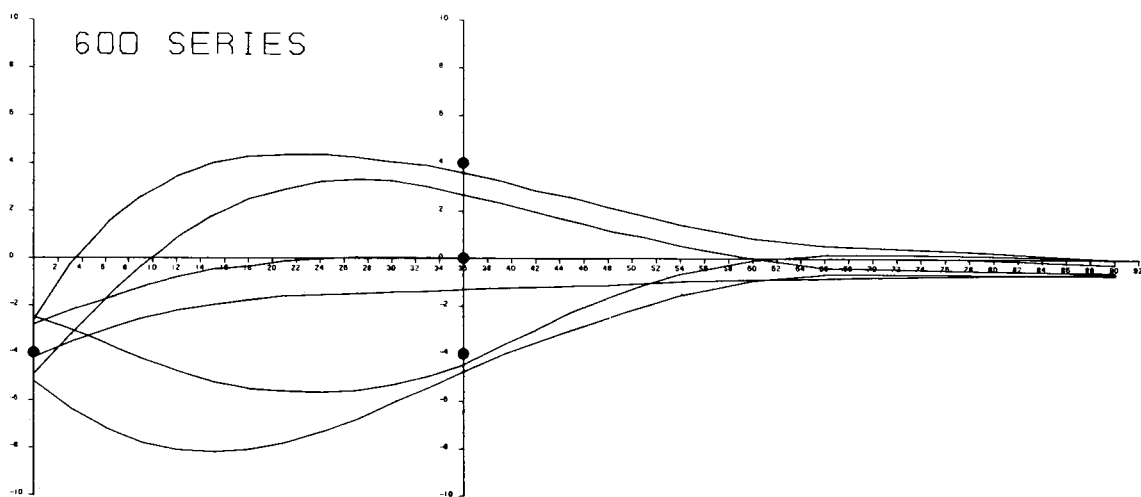
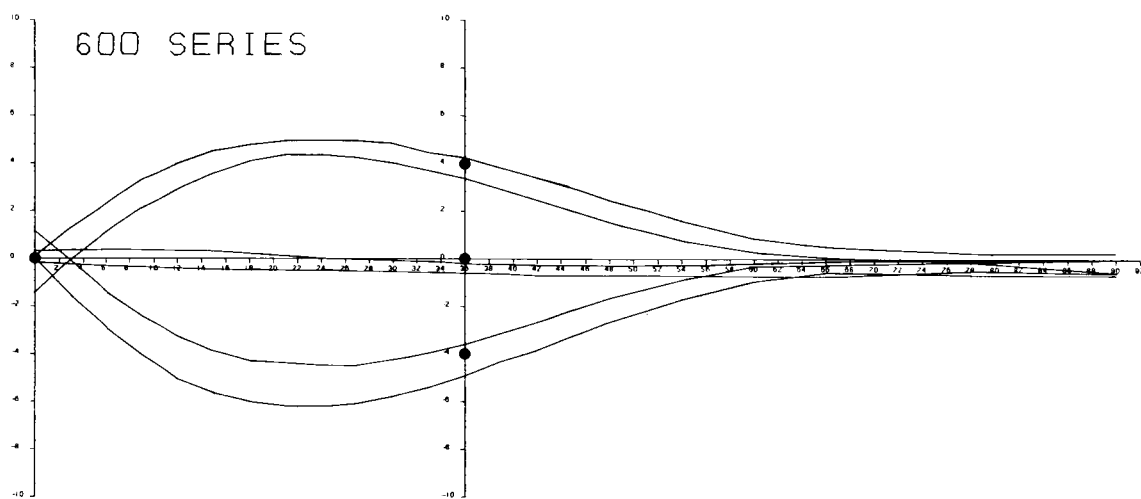
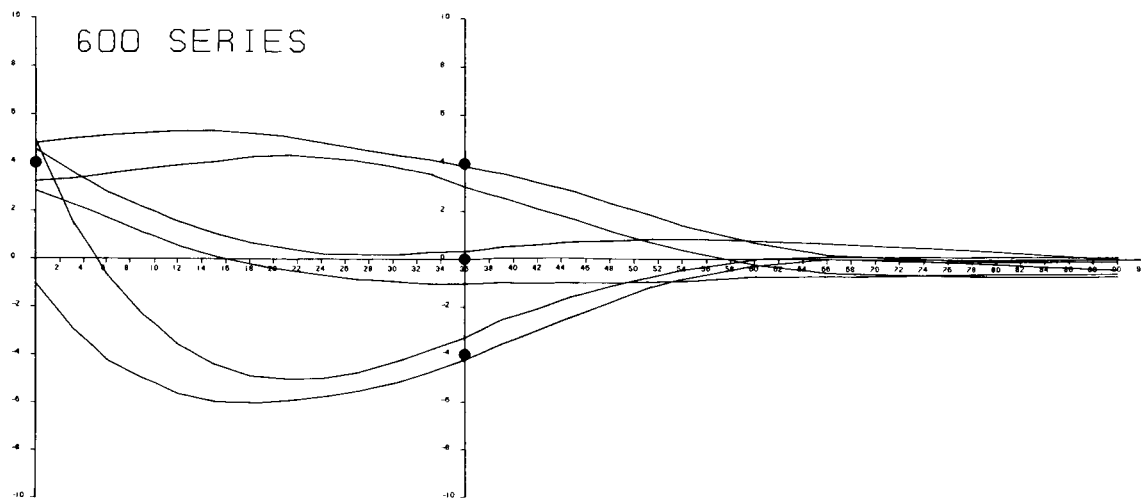
The longitudinal axis of the graph is marked at intervals representing two inches on the track. The origin of the longitudinal axis is the position of the last target. The vehicle moved from right to left. The extreme right hand end of the curve is where the vehicle was when the first target appeared (90 inches short of the origin as shown on the graph). (This apparent reversal in graph convention was due to a mixup in the program to run the computer plotter.) Series 000 is an exception; here the curves start twelve inches past the point of first sight. The latitudinal axis is also marked at intervals representing two inches on the track. However, the scale factor in the lateral direction is one half the scale factor of longitudinal distances. The origin of the latitudinal axis is the center line of the track with positive distances to the right when facing in the direction of the vehicle velocity. The marks on the abscissa of certain graphs indicate the position of critical events not otherwise evident by reference to Figure 7: On Series 500 the mark indicates the vehicle position



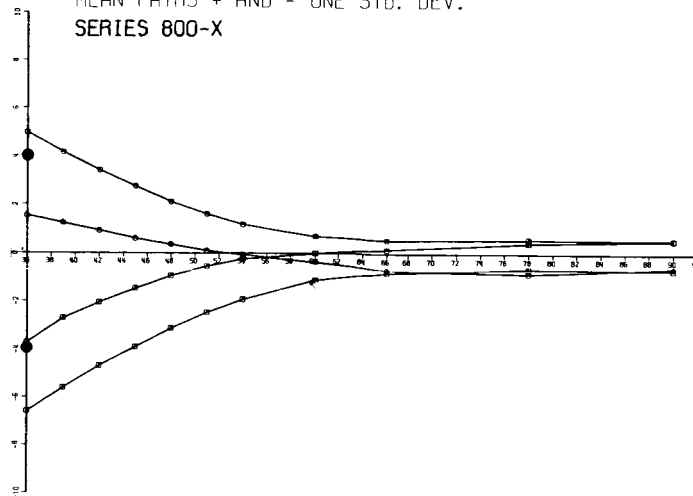
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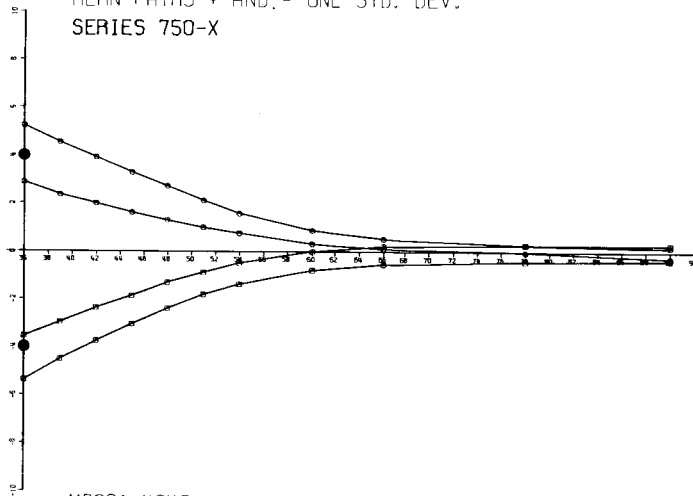


MEAN PATHS + AND - ONE STD. DEV.  
 SERIES 800-X



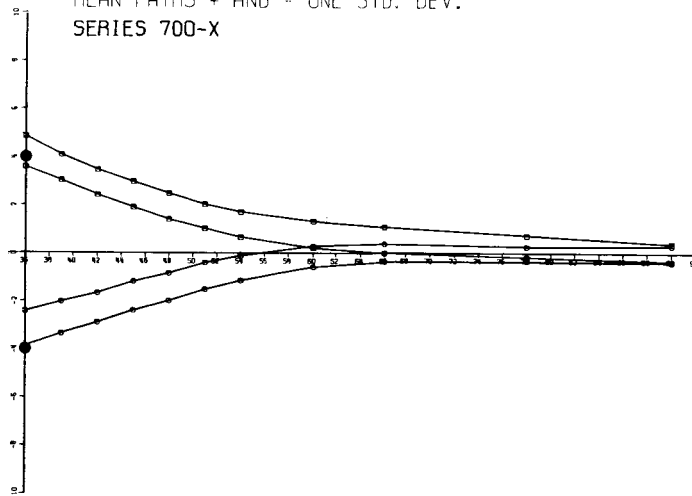
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MEAN PATHS + AND - ONE STD. DEV.  
 SERIES 750-X



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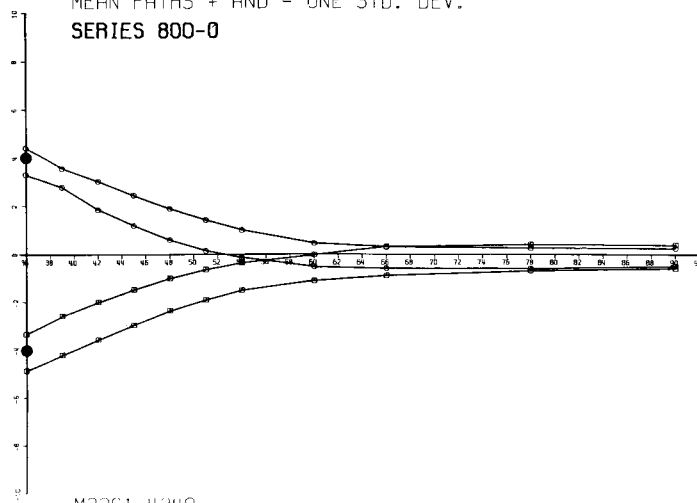
MEAN PATHS + AND - ONE STD. DEV.  
 SERIES 700-X



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MEAN PATHS + AND - ONE STD. DEV.

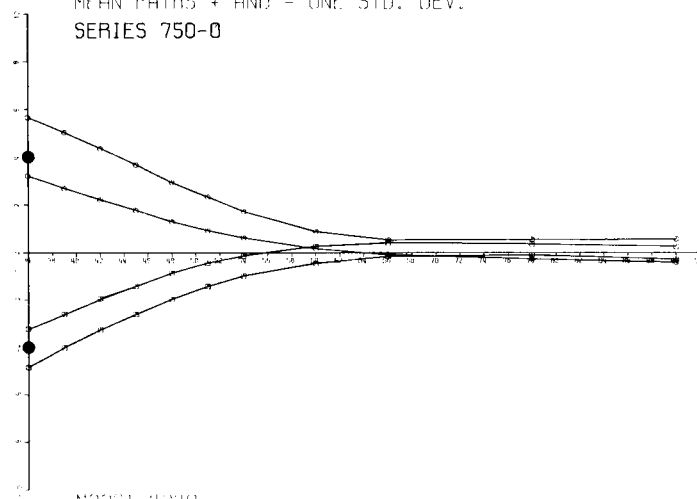
SERIES 800-0



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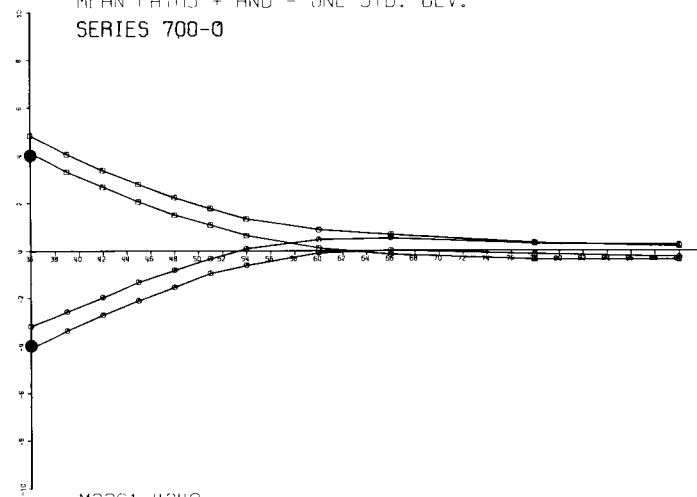
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MEAN PATHS + AND - ONE STD. DEV.

SERIES 700-0



M3261-4348

when the second target appeared; on Series 700, 750, and 800 (both 0 and x) the mark indicates where vision was cut off.

Series and Target Position	df	Variance Ratio	Variance Ratio for 5% Significance
$\frac{200(-4)}{000(-4)}$	4/4	2.07	6.39
$\frac{200(+4)}{000(+4)}$	4/4	2.65	6.39
$\frac{200(-8)}{200(-4)}$	4/4	16.7	6.39
$\frac{200(+8)}{200(+4)}$	4/4	3.56	6.39

Table 2. Variance Ratios for Series 000 and Series 200

#### 4.1 Effect of Increased Number of Alternatives (Increased Information Content of the Stimuli) in Single Target Tasks

A comparison of the graphical results of Series 000 and Series 200 reveals no significant increase in the variance of the trajectories at the position of the target due to the increase in information content of the stimulus. However, Series 200 did indicate an increased variance with farther displacements of the targets from the center line. The variances of the trajectories at the position of the targets eight inches from the center line show a significant difference compared with target positions four inches from the center line, Table 2. It was expected that driving performance would become somewhat more erratic when the subject was responding to targets at increasing distances from his line of travel.

#### 4.2 Effect of Appearance of a Second Target on the Response to First Target

It was expected that the requirement of a second target at the center line, Series 400, might cause an alteration of the response to the first target as compared to that situation where only one target was to appear, Series 000. However, the curves of Series 400 compared with those of Series 000 show no apparent tendency for the driver to alter his course to the first target. Instead of producing a different slope of the trajectory through the first target of 400, the subject seemed to respond after hitting the first target with a relatively large lateral acceleration to return the vehicle to the target on the center line.

#### 4.3 Effect of Increased Number of Positions of Second Target in Two Target Tasks

In the following discussion the abbreviations R, C, and L are used for four inches right of the center line, on the center line, and four inches left of the center line respectively. When enclosed in parenthesis these abbreviations are used to describe a particular target configuration; the first letter is for the first target position and the second letter for the second target position.

The results of Series 300 offer several interesting contrasts to those of Series 000 and Series 400. There are three tasks of the 300 Series, (R,C), (C,C), and (L,C), which are identical to the 400 Series with the exception that the probability of the second target appearing on the center line in the 300 Series was one-third, whereas in the 400 Series this probability was unity. The data show a significant difference in variance at the location of the second target for those tasks of the 300 Series and 400 Series in which the first target was off the center line and the second target was on the center line, (R,C) and (L,C).

In the (R,L) and (L,R) tasks of the 300 Series there was a tendency

for the subject to swing to the inside of the first target; similarly, at the position of the first target the trajectories for those tasks of the 300 Series tended to be inside the trajectories for corresponding tasks of Series 000. Nevertheless, at the position of the first target the means of these Series 300 runs were not significantly different from the location of the center of the target.

In the (L,R) and (R,L) tasks of the 300 Series it might have been expected that to avoid large external accelerations between the first and second targets the subject would initially accelerate to a lateral position beyond the first target then attempt to cross both targets in a straighter path. However, instead of crossing the first target at a slope which is directed back toward the center line, the trajectories are divergent at the position of the first target. Moreover, up to the position of the first target, there is no apparent difference in the results of those runs in which the second target was on the opposite side of the center line and those in which the second target was on the same side of the center line as the first target. The trajectories become parallel to the center line at approximately one-third to one-half the distance from the first to the second target.

Wide confidence limits are particularly noticeable in those tasks of the 300 Series in which the second target was on the center line. The variance for these tasks is significantly greater than those seemingly more difficult tasks in which the driver had to cross the centerline to hit the second target. One explanation for this phenomenon may be that in attempting the maneuver in which the targets are on the opposite sides of the center line the driver is required to turn as fast as possible to reach the second target. Providing the driver could consistently reproduce this rate-limited open loop response his variability would be reduced. By contrast the task of returning from off the center line to hit a target on the center line may be a critical combination of greater maneuverability and a task requiring closed loop

control. A possible explanation for the degraded performance is that it is a task in which the driver still has more lateral acceleration available than he is putting to use.

#### 4.4 Effect of Delaying Second Target in Two Target Tasks

Delaying the appearance of the second target had the effect of reducing the variance at the second target. The confidence limits for the trajectories of the 500 and 600 Series, in which the second stimulus was delayed, are generally much smaller than for the corresponding tasks of the 300 Series in which the target appeared simultaneously.

The improved response of the tasks in which the second target was delayed, Series 500 and 600, suggests that the driver is confused by the simultaneous occurrence of the two targets. Rather than simultaneously viewing both targets and then carrying out an integrated response to both targets, he appears to view the first, respond to the first, view the second, respond to the second. One might hypothesize that he treats the second after at least one psychological refractory period, but then with some confusion.

The most noticeable difference between the 500 Series and the 600 Series is that the trajectories of the 600 Series show a prominent left bias. This tendency for the trajectories to be offset to the left is evidenced by the mean of the trajectories being significantly to the left of the center of the first target when the second target was to the left of the center line. Also, the (L,L) trajectories of Series 600 swing approximately two and one-half inches to the left of the target center, whereas the (R,R) trajectories are almost parallel to the center line between the first and second targets. There is no significant difference in the variances of the corresponding tasks of the 500 Series and 600 Series at the location of the second target.

#### 4.5 Effect of Target Width and Viewing Time in Single Target Tasks

Changing the width of the targets results in no noticeable difference in the variance or shape of trajectories. Variability for the

one-half inch wide target in the 700-0, 750-0, and 800-0 series was not smaller than that for the wider target, 700-x, 750-x, 800-x, a possibly unexpected result for a subject driving closed loop.

More supportive of the open loop hypothesis is the fact that there is no significant difference between the occluded vision tasks and the total vision tasks until the viewing time was reduced to .12 seconds. This time is approximately one-tenth the time from the point of first sight until the target was hit. Since in the 800 Series the view was terminated about .38 seconds before any lateral motion of the vehicle is detected it was impossible for the subject to make path corrections based on present error information as the vehicle traveled toward the target. In the 750-x Series the viewing period was approximately .38 seconds and the view was terminated about .12 seconds before lateral motion was detectable, again making closed loop operation impossible during the occluded portion. However, the difference in variance between these trajectories and those run concurrently with total view is hardly noticeable.

## 5. DESCRIPTION OF EXPERIMENTAL APPARATUS AND PROCEDURE FOR COMPUTER GENERATED OSCILLOSCOPE DISPLAY SIMULATION

The oscilloscope display experiment was an abstract simulation of essentially the same driving task considered using the T.V. remote controlled car simulator.

The oscilloscope was driven by a PDP-8 digital computer. The computer was programmed to display three points on the scope. Two of the points were stationary targets and the third suddenly appeared at the bottom and moved upward on the scope face with constant vertical velocity of the moving point, Fig. 8.

The hand controller which the subject used was a modified high speed telegraph key or "bug". The computer effected a positive or negative pulse of lateral acceleration, i.e. a change in velocity, each time the toggle arm of the bug was pressed to the right or left.

Digital computation does not allow continuous variation of the position and velocity which define the states of the targets and the moving point. Thus in reality the display consisted of a grid space with 64 discrete space states in both vertical and horizontal directions.

Accelerations could occur only at defined space states. The program permitted only a single unit change in velocity at each vertical space state.

The display program allowed the two targets to be placed at any position defined by the 64 x 64 grid space. The initial vertical position of the moving point was always at the bottom edge of the display, but the initial horizontal position could be preset to any of the 64 horizontal states.

The vertical velocity of the moving point ranged from 5 to 25 grid widths per second. If the oscilloscope gain was set such that the distance between the first and sixty-fourth vertical states was 20.5 centimeters, then the vertical velocity could be set between 1.6 and 8.0

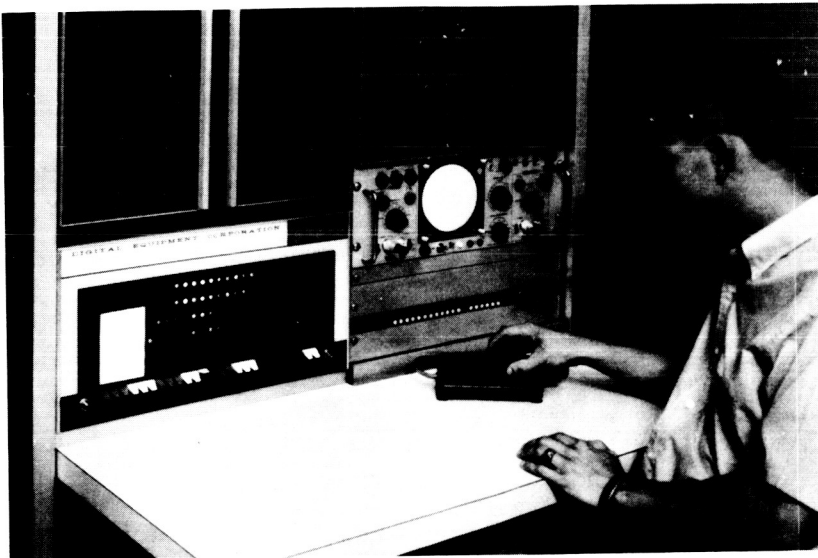


Figure 8-a. Time Lapse Photo of Subject Making Test Run on Scope Experiment

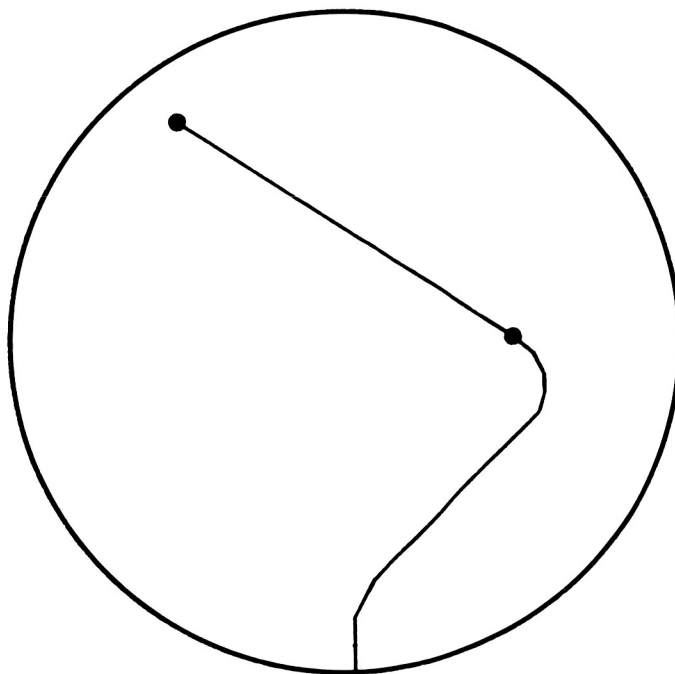


Figure 8-b. Optimal Trajectory for Target Configuration Shown on Scope Above and Cost Function of Equation 2.

centimeters per second.

The display was terminated when the moving point reached the terminal vertical state. This terminal state could be preset to any of 64 vertical states. At the end of each run a score was computed. The score was equal to the sum of the absolute values of the horizontal distances by which the first and second targets were missed by the moving point multiplied by a constant which was preset by the experimenter, plus the sum of the unit horizontal accelerations used in maneuvering the moving point from start to finish.

$$\text{SCORE} = K [(XT1 - X_{Y=YT1}) + (XT2 - X_{Y=YT2})] + \sum_{n=3}^{n=YMAX} (X_n - 2X_{n-1} + X_{n-2})$$

.....Eq. 1

where: K = importance weighting function on missing targets

X = horizontal space state of moving point

Y = vertical space state of moving point

XT1, YT1 = space state of first target

XT2, YT2 = space state of second target

YMAX = terminal vertical space state

At the end of each run the score for that run was shown to the subject via the oscilloscope.

In each run of the oscilloscope experiment two targets appeared. Each target had the same vertical space state throughout the entire series of runs. There were three possible horizontal space states at each of the two vertical states, resulting in nine different target situations. Figure 9-a details the location of the targets and the starting location of the moving point. An entire set of runs consisted of five runs for each target situation at each of three speeds, 1.6, 3.2, and 4.8 centimeters per second, resulting in 135 runs total. The

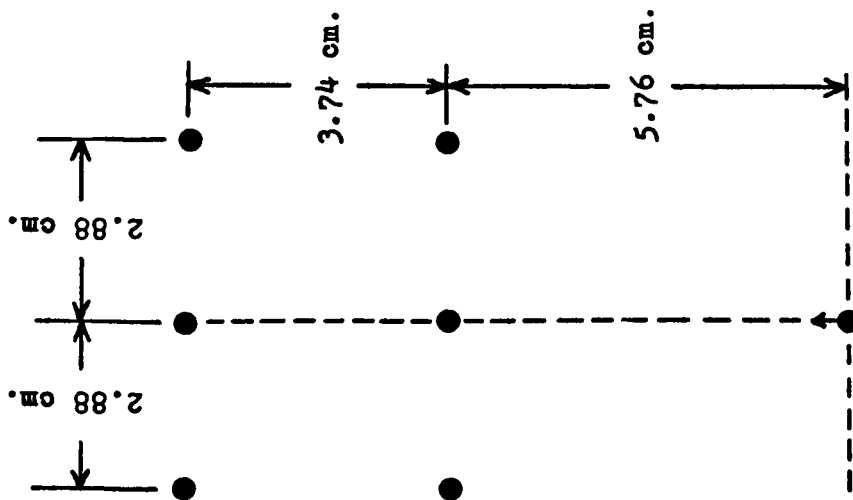


Figure 9-a. Target Configuration for Scope Experiment

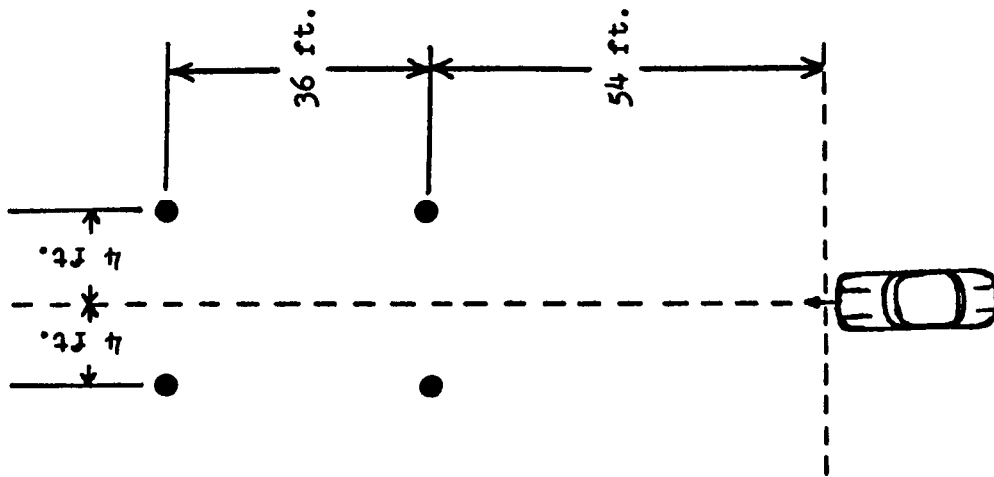


Figure 9-b. Target Configuration for Automobile Experiment

three parameters of speed and horizontal target positions were varied randomly.

Three subjects were used in the oscilloscope experiment. The set of 135 runs took approximately forty minutes to complete. Each subject completed the entire set of runs on each of four different days. The first day was practice and the trajectories were not recorded. The subject's instructions were to minimize his score. The score was computed from Equation 1, with the coefficient K set equal to 2. With this importance weighting on error, it was necessary for the subject to hit the target in order to minimize his score, while trying his best to economize on acceleration.

$$\text{SCORE} = 2[(Y_{T1} - X_{Y=Y_{T1}}) + (X_{T2} - X_{Y=Y_{T2}})] + \sum_{n=3}^{n=Y_{MAX}} (X_n - 2X_{n-1} + X_{n-2})$$

.....Eq. 2

## 6. RESULTS OF COMPUTER GENERATED OSCILLOSCOPE DISPLAY SIMULATION

The results of the (L,R) and (R,L) tasks are shown in graphical form in Figures 11, 12, and 13. Mean trajectories are shown for (L,R) and (R,L) target configurations for three speeds by each subject. Also the trajectory of the optimal path for the cost function of Equation 2 is shown on each graph. The optimal control model is described in Appendix II.

Of the nine target configurations, data was reduced for only the two most difficult tasks of the computer generated oscilloscope display simulation experiment. The mean trajectories of Figures 11, 12, and 13 represent fifteen runs each. The means and standard deviations of the scores from Equation 2 corresponding to these trajectories are shown in Table 3. The trajectories approaching the first target are always nearer the center for higher speeds. This phenomenon, probably an effect of the subjects' reaction times when the display first appeared, is obvious in the results of all three subjects.

The mean trajectories of the three subjects tend to hit the first target. However, for the second target the subjects generally pass on the inside. Subject LLE was not familiar with the optimal trajectory for minimizing score. Subject DJB was familiar with the shape of the optimal trajectory; however, his overall mean score was higher than subject LLE. It is interesting to note that the mean trajectories of subject DJB are closer to the optimal path than those of subject LLE. Nevertheless, due to subject DJB's greater variability, as indicated by score variances, his overall score was higher than that of subject LLE. The third subject (RDR) was an author, who, obviously, was familiar with the optimal trajectory. Because the parameters for the sequence of 135 runs were randomly varied and stored in the memory of the computer, it was impossible for the author to predict the tasks in advance. Subject RDR had considerably more training than either of the other two subjects. Although subject RDR's intention was to minimize

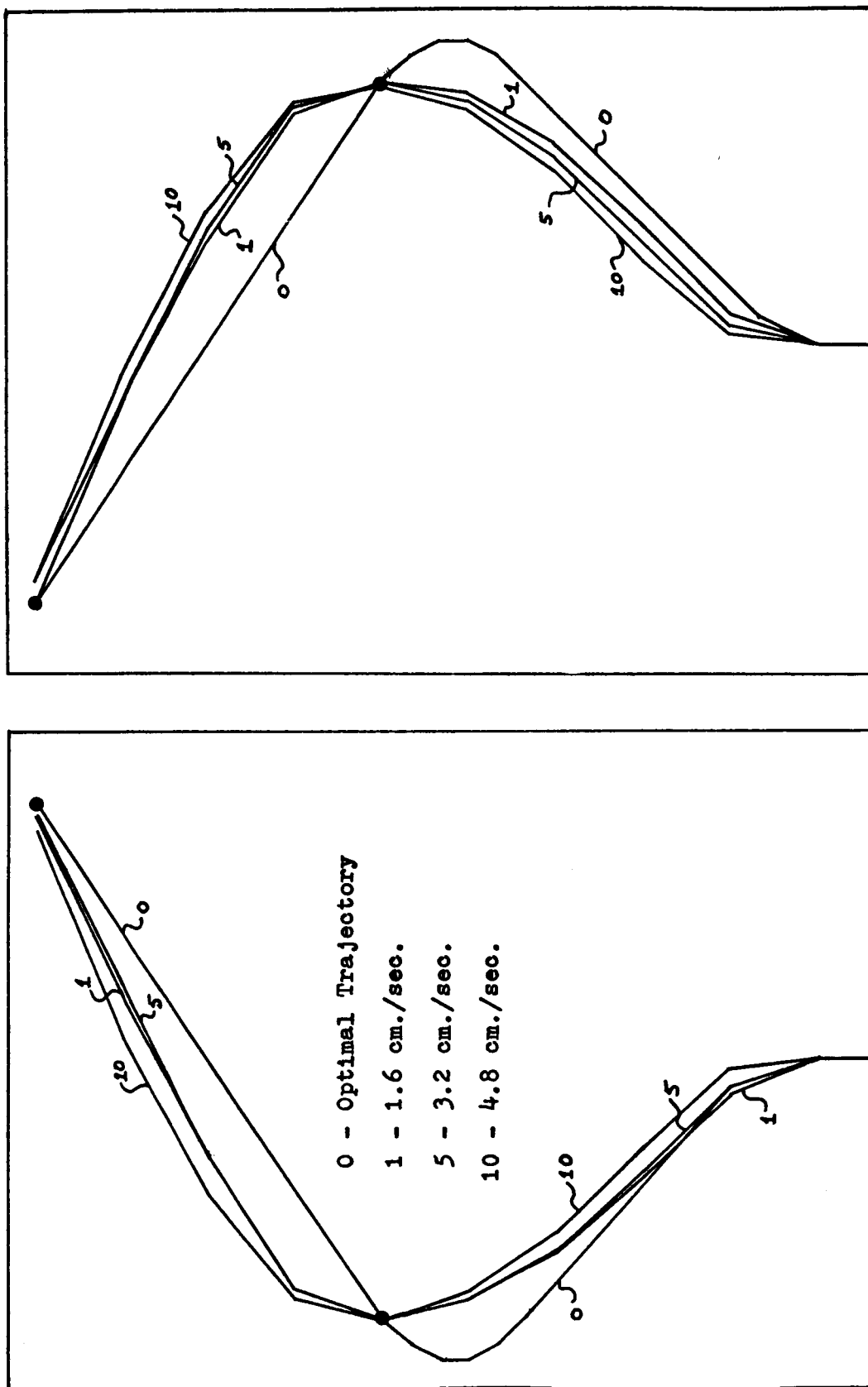


Figure 10. Graphical Results from Computer Generated Oscilloscope Display Simulation, Subject LLE

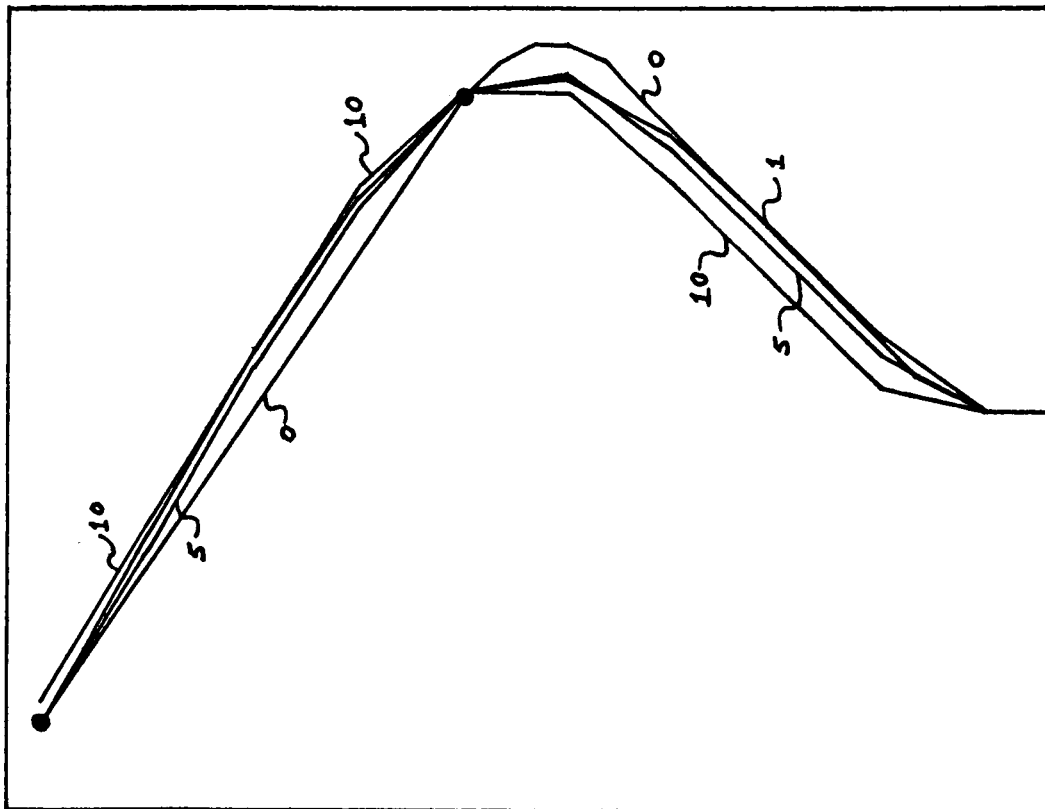
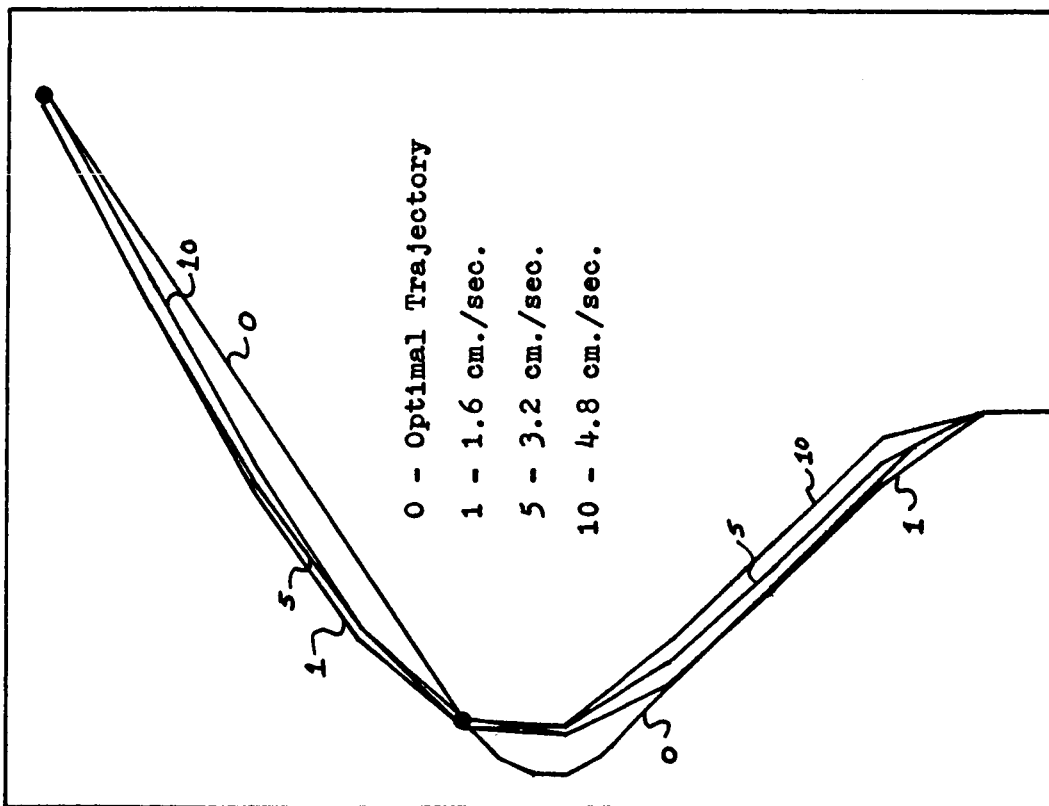


Figure 11. Graphical Results from Computer Generated Oscilloscope Display Simulation, Subject RDR

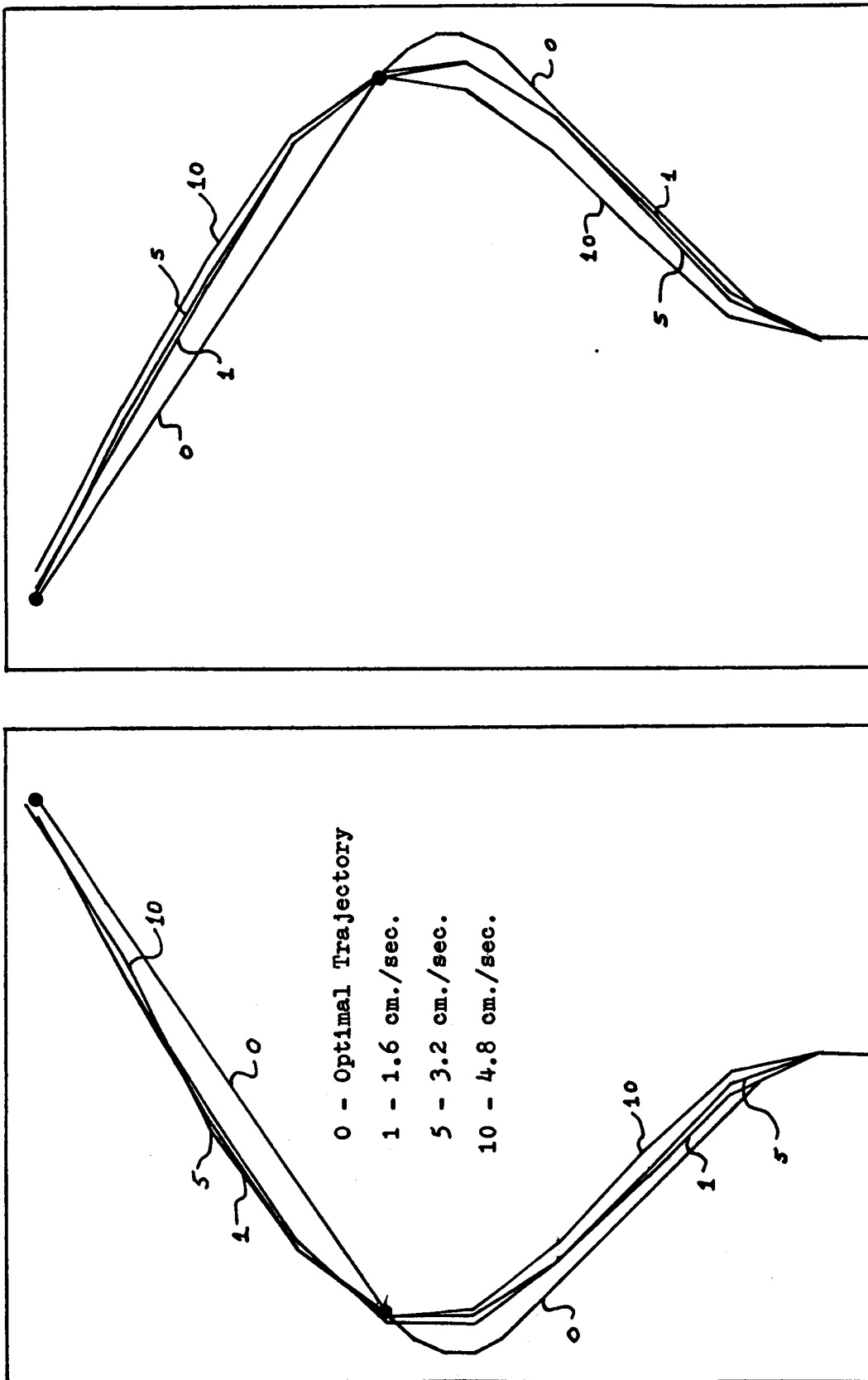


Figure 12. Graphical Results from Computer Generated Oscilloscope Display Simulation, Subject DJB

his overall score, he deliberately attempted to follow the path dictated by the optimal control model for minimum score. With training, subject RDR succeeded in reducing his variability and in approaching the optimum trajectory, thus reducing his overall mean score. The effect of training in these tasks is more evident when considering the fact that all three subjects got significantly lower scores on subsequent days.

Subject	Target Position	Speed 1.6 cm/sec.	Speed 3.2 cm/sec.	Speed 4.8 cm./sec.	Grand Mean
LLE	LR	12.5 (2.5)	12.4 (2.5)	14.0 (3.5)	13.3 (3.1)
	RL	13.0 (2.3)	13.9 (3.7)	14.1 (3.4)	
DJB	LR	12.1 (3.7)	13.1 (4.5)	14.9 (5.8)	13.8 (5.3)
	RL	12.8 (2.9)	12.9 (3.2)	16.8 (8.6)	
RDR	LR	11.2 (2.9)	10.3 (1.6)	13.4 (3.4)	11.3 (2.8)
	RL	10.2 (2.1)	10.6 (2.0)	12.1 (3.4)	
Optimal	either	7.0 (0)	7.0 (0)	7.0 (0)	7.0 (0)

Table 3. Mean Scores for Computer Generated Oscilloscope Display Simulation; ( ) Represent Standard Deviation

These mean trajectories of the scope display simulation are fundamentally different in shape from those corresponding trajectories of Series 300 of the T.V. remote controlled car simulation. In particular the position of zero slope (relative to the center line) of the trajectories of the scope simulation occurs on or before crossing the position of the first target. By contrast in the T.V. remote controlled car simulation, the position of zero slope was from one-third to one-half the distance from the first to the second target.

## 7. DESCRIPTION OF EXPERIMENTAL APPARATUS AND PROCEDURE FOR AUTOMOBILE EXPERIMENT

The experiment using the real car was designed to be a scaled-up task corresponding to the driving simulator. The tests using the full scale automobile were carried out in an asphalt surfaced parking lot, measuring six hundred feet in length and one hundred feet in width. The car used was an eight cylinder 1966 Chevrolet with original equipment tires, power steering, and automatic transmission.

Two white targets (8" x 8" x 4" high) were placed either four feet to the right or four feet to the left of the center line, respectively at fifty-four and ninety feet from the point where the subject first saw the targets, Figure 9-b.

The procedure for each run was as follows: the subject would drive the car to the end of the parking lot and position it so that it was in line with the center line through the target area. The sun visor was pulled down obscuring the driver's view except for a few feet directly in front of the car. Once the subject could not see that target area a third person would position the targets for the next run. When the ready signal was given by the experimenter, the subject would quickly accelerate the car to a speed of thirty-five miles per hour. The experimenter, sitting in the rear seat directly behind the subject and looking around the sun visor, would steer the car until the subject could see the center line, visible below the sun visor. At a distance of fifty-four feet from the first target the experimenter would flip the sun visor up and the subject would attempt to steer the center of the car over the targets. As the car was returning to the end of the parking lot for the next run the third person would record, to the nearest inch, the track of the right front wheel of the car at the last target and at four eighteen foot intervals in front of the last target. The speed was intended to be constant at thirty-five miles per hour. However, for situations with the targets on opposite sides of the center line the speed had

usually dropped to about thirty-two miles per hour upon crossing the second target. The maneuvering required for hitting targets on opposite sides of the center line was drastic enough to cause frequent side-slipping of the rear wheels. However, the subject never lost control of the car. At forty miles per hour the experimenter found that the car would completely spin out when attempting this target setup.

A total of ninety-two runs were taken on three mornings. The first twenty-eight runs, taken on the first morning, were considered practice and not used as data runs. The sixteen data runs for each target configuration were presented in random order.

## 8. RESULTS OF AUTOMOBILE EXPERIMENT

The subject for the automobile experiment was the same as for the T.V. remote controlled car simulation experiment. The graphical results, Figure 13, from the automobile experiment show a tendency for the subject to hit the inside of the center of the second target for the (R,L) target configuration. However, the trajectory mean is not significantly different from the center of the target at the position of the second target.

The results show a significant difference in variance at the position of the second target for the (R,L) and (L,R) target configuration as compared to the corresponding tasks, Series 300, of the T.V. remote controlled car simulation experiments. The location of the position of zero slope (relative to the center line) is approximately at the position of the first target. This occurrence of zero slope is considerably sooner than in the corresponding two target tasks of the T.V. remote controlled car simulation experiments.

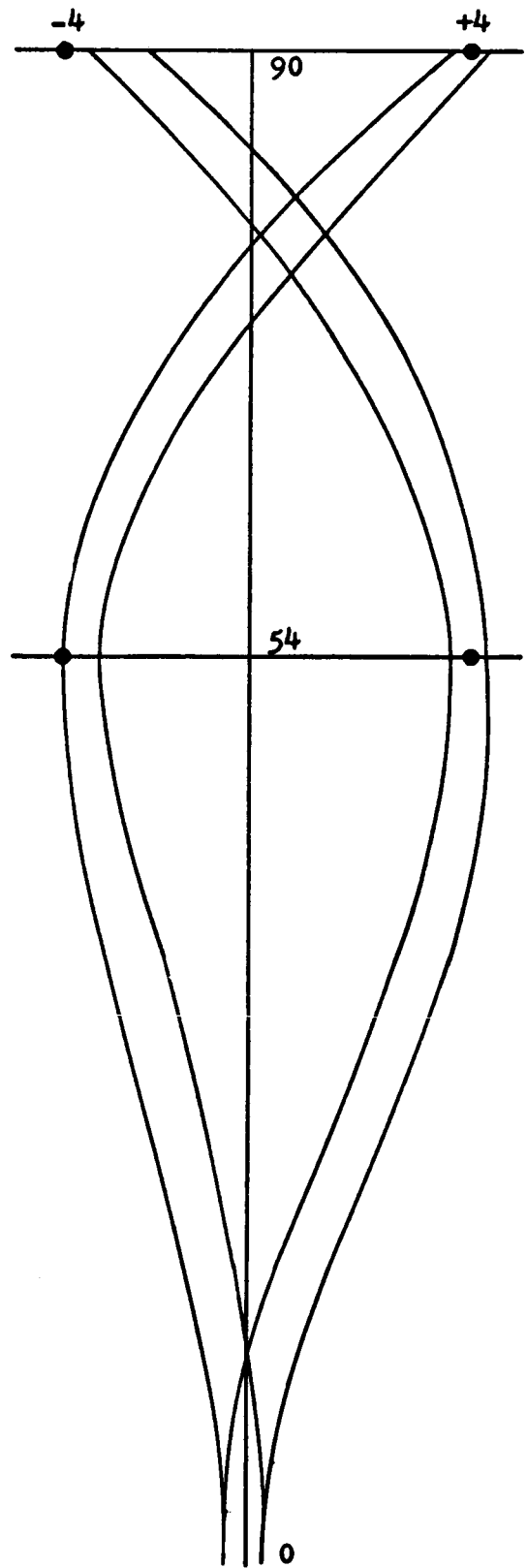
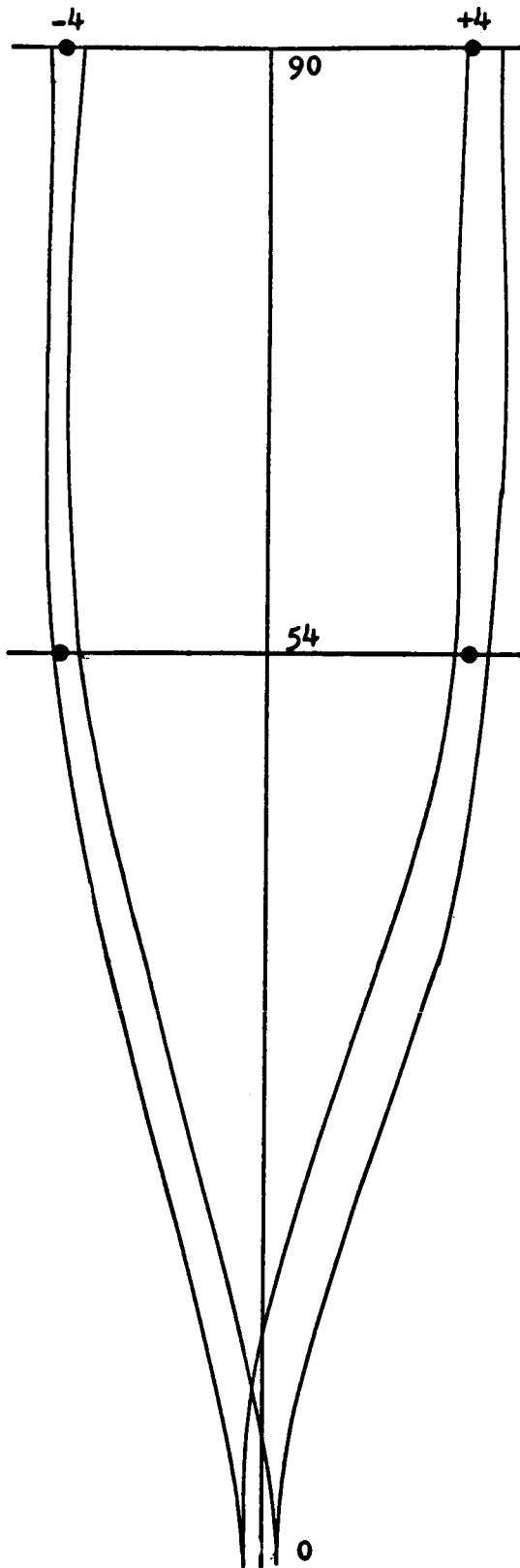


Figure 13. Graphical Results from Automobile Experiment

## 9. CONCLUSIONS

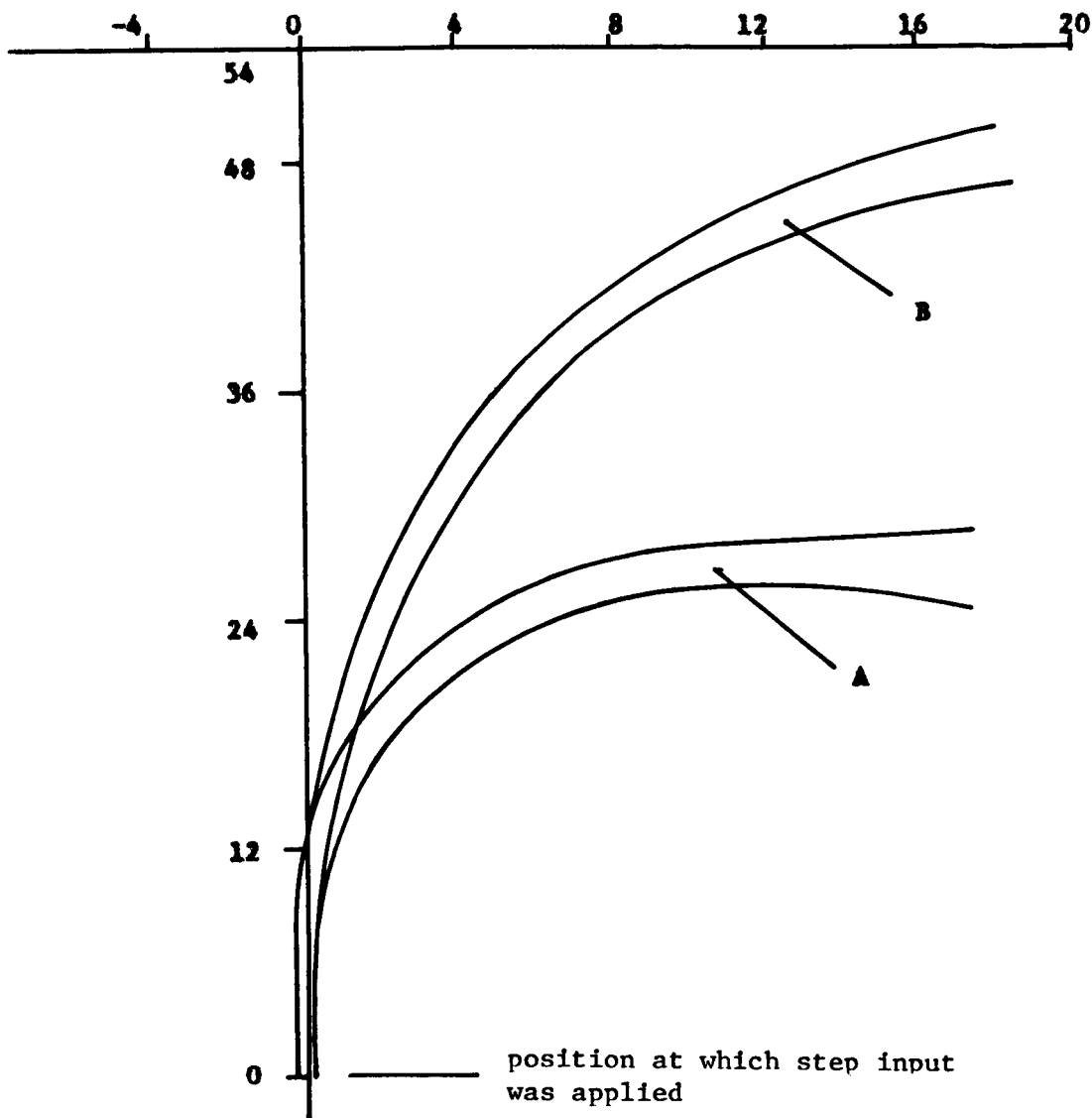
The results of two driving simulation experiments, one abstract and the other visually realistic, have been presented with the results of an actual automobile experiment for similar control tasks. Comparisons of the results of the three experiments reveal small but consistent differences in the control tendency and variability of the trajectories for similar control tasks:

1. Results with the T.V. remote controlled model car simulator showed that the subject responded later than in the computer oscilloscope display or in the actual car experiment. With both the actual car and T.V. simulation he tended to respond to each target as it came. By contrast in the less realistic or more abstract scope display experiment the subject tended to "get lined up" ahead of time with respect to the line connecting the two targets. These differences may have been due to the fact that the scope targets were not seen in perspective whereas the model T.V. and actual car targets were necessarily viewed in perspective. Further, the T.V. simulator constrained peripheral vision. In general the subjects consistently differed from optimal control performance based on a criteria implied by the subjects' instructions, the difference being characterized by insufficient planning for the second of two targets until after engagement with the first. With training there was some trend in the direction of optimal performance.
2. Of the three experiments inter-run variability was greatest in the T.V. model car simulator, least in the scope simulator. Variability was usually greater for the second of two targets, though the variance ratio was not statistically significant ( $>6.39$  for  $df = 4/4$ ).
3. In the T.V. simulator when two spatially ordered targets were presented simultaneously the variability of response to the second

target was greater than when presentation of the second target was delayed by one half second. Again the variance ratios were not statistically significant ( $>6.39$  for  $df = 4/4$ ).

4. Some results for experiments in which the driver was forced to steer for one target (of two possible positions) in open-loop fashion (without any visual feedback) after a single glance at the actual target position showed surprisingly little difference from the closed-loop results.

The effects noted above will require allocation of greater resources to verify statistical significance.



#### APPENDIX I. STEP RESPONSE OF REMOTE CONTROLLED CAR

Curves A and B represent the confidence limits of seven and six runs respectively for a step input to the servoamplifier. Curve A represents the response to an equivalent step input to the steering wheel of a complete center to stop clockwise turn (360 degrees). Curve B is the response of an equivalent 90 degree step input to the steering wheel. The confidence limits are one standard deviation.

## APPENDIX II. PDP-8 COMPUTER PROGRAMS

The programs of the optimal control model utilizing the dynamic programming algorithm and of the oscilloscope display simulation are written in PAL 3 assembly language for a Digital Equipment Corporation PDP-8 digital computer. The PDP-8 is a high speed computer with 4096 twelve bit words of core memory.

The program of the optimal control model allows the experimenter to type in: initial starting position, vertical and horizontal coordinates for two targets, maximum vertical and horizontal dimensions of allowable space, error weighting function, velocity cost function, and acceleration cost function. After computation has been completed for a set of parameters the optimal trajectory is displayed on the oscilloscope. Options are provided for typing out the optimum score and trajectory and for inputting new parameters. The program requires approximately  $1200_{10}$  words of memory.

Because of their length these programs are not included. However, the programs are maintained by the Man Machine Systems Laboratory of the Mechanical Engineering Department, M.I.T.

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